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CONTRIBUTION TO THE THEORY OF SURFACE PHENOMENA OF POROUS BODIES OCCURRING DUE TO HEAT AND MASS TRANSFER PROCESSES

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Abstract—A mathematical model is developed for the solution of heat and mass transfer problems of porous media in a non-steady state. The analysis of the wet bulb temperature in the steady state is derived from a new explanation of the w.b.t. in the non-steady period, together with the physical parameters determining it. Accordingly the Lewis factor gives a proper interpretation in the state in question. In addition a new interpretation of the Lewis rule is given for the non-steady state.

NOMENCLATURE

- c, specific heat [J kg^{-1} K^{-1}];
- G, mass velocity of the carrier gas [kg s^{-1}];
- h, heat transfer coefficient between the gas and the sorptive body [J kg^{-1} s^{-1} m^{-2} K^{-1}];
- i, I, enthalpy of gas and solid, respectively [J kg^{-1}];
- k, mass transfer coefficient between the sorptive body and the gas [kg m^{-2} s^{-1}];
- M, mass velocity of the solid [kg s^{-1}];
- q', defined by equation (9) [J kg^{-1}];
- q'', differential sorption energy, defined by equation (9a) [J kg^{-1}];
- R, integral sorption energy [J kg^{-1}];
- R_g, gas constant;
- r, specific evaporation heat [J kg^{-1}];
- t, ϑ, temperature and its difference [°C];
- t_w, wet bulb temperature [°C];
- X, absolute moisture content of solids;
- x, absolute humidity of gas;
- z, difference of absolute humidities of gas.

Greek symbols

- η, relative mass flow rate.

Subscripts

- a, gas;
- A, air;
- e, equilibrium;
- m, material;
- 0, limiting or initial state;
- v, superheated vapour;
- w, liquid (water), wet bulb.

Dimensionless number

- L, Lewis factor, $h/(kc)$.

INTRODUCTION

THE EQUILIBRIUM rules of liquid and gas at the interface can generally be unambiguously characterized and the parameters of the equilibrium state be determined. The same applies to the equilibrium of liquid and gas bound to solid matter.

The present paper deals with the equilibrium conditions of simultaneous heat and mass transfer between a gas, a liquid, and a solid phase which interact by sorptive bonds, and confine the transfer to isothermic and convective flow.

BASIC EQUATIONS

Let us place in unit time a sorptive material of mass-flow M into a gaseous medium of mass-flow G which has a constant temperature t_A and absolute humidity x_A.

The ratio of the mass velocities is accordingly

$$\eta = M/G = \text{const.} \tag{1}$$

i.e. the relative mass flow of the dry solid related to the dry gas is constant.

At an intermediate temperature, t, a moisture content X is bound to unit mass of solid and a humidity x to that of gas. This corresponds to a new wet bulb temperature of the gas flow.

The non-steady state involves transfer of heat from the air to the material, resulting not only in the simple evaporation of water but also in the gradual liberation of the sorptively bound moisture. The momentary equilibrium takes place at the temperature of the material, which meanwhile rises to some value t. The material has a moisture content X which is bound to unit mass of the solid. The compensation temperature of the air, t_w, is the same temperature value i.e. another wet bulb temperature differing from t_0 while its absolute humidity is some $x = x_A + z$ (Fig. 1).

For the enthalpies of gas and solid, momentarily being in equilibrium at the compensation temperature $t = t_w$, we can write [1, 2]

$$i = r_0 x + (c_a + c_v x)t = r_0 x + ct, \tag{2}$$

$$I = (c_m + c_w X)t + R(X) = Ct + R. \tag{3}$$

The principle of conservation of enthalpy and mass

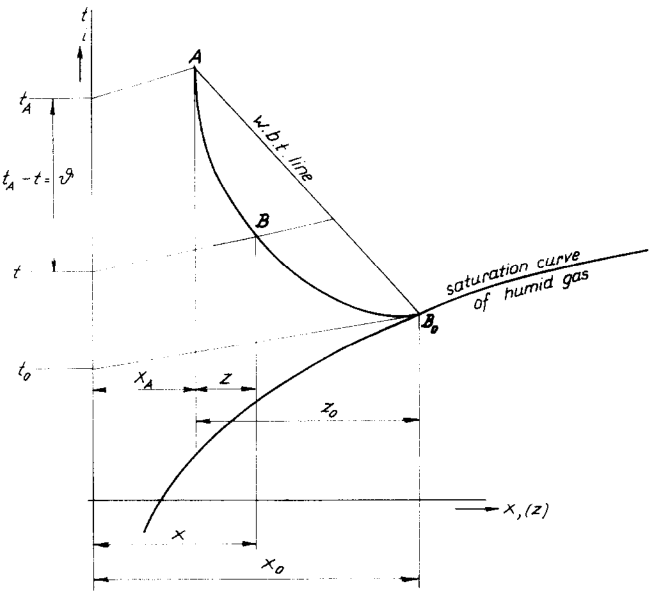

FIG. 1. State change curve of humid gas in the non-steady stage.

enables relations for the change of state of the non-steady stage to be obtained.

The enthalpy changes may be expressed by the equation

$$\mathrm{d}i = \eta\, \mathrm{d}I \tag{4}$$

and continuity of mass results in

$$\mathrm{d}x = \eta\, \mathrm{d}X \tag{5}$$

and so we obtain

$$\frac{\mathrm{d}i}{\mathrm{d}x} = \frac{\mathrm{d}I}{\mathrm{d}X}. \tag{6}$$

Substituting the values of equations (2) and (3) into equation (6) results in [3–6]

$$\frac{\mathrm{d}i}{\mathrm{d}x} = c\frac{\mathrm{d}t}{\mathrm{d}x} + r + t, \tag{7}$$

$$\frac{\mathrm{d}I}{\mathrm{d}X} = C\frac{\mathrm{d}t}{\mathrm{d}x} + t - q' - q'' \tag{8}$$

where the values of q' and q'' are expressed as [7]

$$q' = -C\frac{\mathrm{d}t}{\mathrm{d}X}, \tag{9}$$

$$q'' = -\frac{\mathrm{d}R}{\mathrm{d}X}. \tag{9a}$$

From equations (7) and (8), with equations (9), (9a), (4) and (6),

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{\mathrm{d}t_w}{\mathrm{d}x} = \frac{r + q''}{\eta C - c} \tag{10}$$

and

$$\frac{\mathrm{d}t}{\mathrm{d}X} = \frac{r + q''}{C - (c/\eta)}. \tag{11}$$

Since from equations (4) and (6) (Fig. 1)

$$\eta = \frac{z_0}{X_0 - X_e} \quad \text{and} \quad \frac{z}{z_0} = \frac{X - X_e}{X_0 - X_e}$$

it follows that:

$$X = X_e + \frac{z}{\eta} \text{and } x - x_A = \eta(X - X_e). \tag{12}$$

In equations (10) and (11) the term C may be expressed as

$$C = c_m + c_w X = (c_m + c_w X_e) + (z/\eta)\, c_w = C_e + (z/\eta)\, c_w \tag{12a}$$

and

$$c_A = c_a + c_v x_A.$$

From equation (10)

$$\frac{\mathrm{d}t}{\mathrm{d}z} = \frac{r + q''}{z(c_w - c_v) + C_e \eta - c_A}. \tag{10a}$$

Equation (11) represents the differential equation of the function $t = t(X, \eta)$ [7] which is the temperature curve of the non-steady state change of the material and where η, the relative flow rate, is constant in one and the same process and may therefore be regarded as a parameter. In equation (11) the variables are not separated, and thus the integration is feasible by numerical computation only.

The $t(X)$ curve is plotted in Fig. 2. It shows that with decreasing X, and therefore increasing q'', the curve becomes steeper. In the case of

$$X = \infty, \quad \left.\frac{\mathrm{d}t}{\mathrm{d}X}\right|_{\infty} = 0,$$

according to equation (11), an increasing η (marked

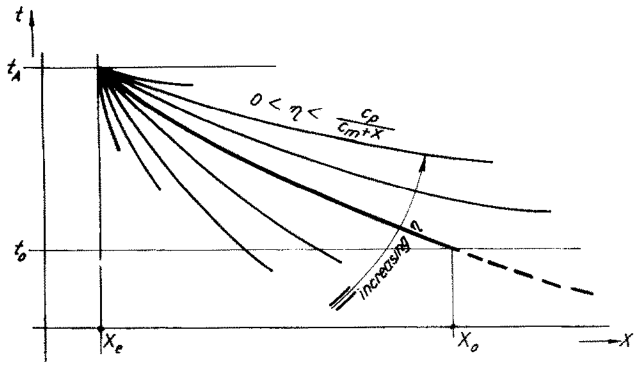

FIG. 2. Material temperature curve vs X.

with an arrow) gives curves that start from A and decrease with X less and less rapidly.

APPLICATION TO A SIMPLIFIED MODEL

Let us place a moist body in an air flow of temperature t_A. In the momentary equilibrium state, characterized by the compensation temperature $t = t_w$, the temperature difference is $t_A - t = \vartheta$.

The heat transmitted due to this temperature difference is used for the evaporation of water, for the warming of solid material and also to compensate for the sorption energy. We can write, per unit surface area of moist body [6, 7],

$$kz\,(r + q' + q'') = h\vartheta. \tag{13}$$

We define now the dimensionless number L

$$L = \frac{1}{c}\frac{h}{k} \tag{14}$$

and from this combined with equation (13), ensues

$$z(r + q' + q'') = cL\,\vartheta. \tag{15}$$

After substituting the value q' from equation (9) we obtain

$$q' = C\eta\frac{\mathrm{d}t}{\mathrm{d}z}$$

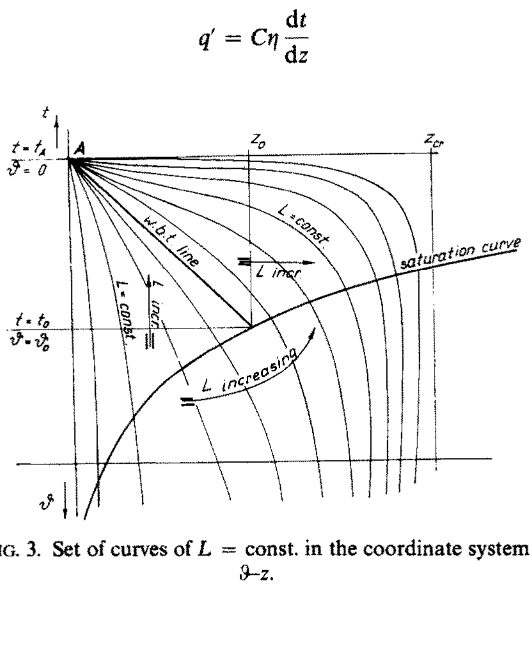

FIG. 3. Set of curves of L = const. in the coordinate system ϑ–z.

and

$$\frac{\mathrm{d}t}{\mathrm{d}z} = \frac{(r + q'') - cL(\vartheta/z)}{C_e\eta + c_w z}. \tag{16}$$

Combining equations (10a) and (16)

$$L = (r + q'')\frac{1}{c\vartheta}\frac{z}{[1 - (c_w z + C_e\vartheta)/(c_v z + c_A)]}. \tag{17}$$

Equation (17) expresses the dependence of L on z and t. As $z = z(t)$ is determined by equation (10a), so equation (17) means that at every point of the function (10a) the associated value of L can be obtained.

On the other hand equation (17) is suitable, too, to plot the L = const. values corresponding to the various ϑ values.

This enables the interpretation and depiction of the set of curves of L = const. in the coordinate system ϑ–z. The set is expressed by the function $\vartheta = \vartheta(z, L =$ const.). Figure 3 represents the set of L = const. curves. The saturation curve is also shown and on the abscissa z the value of

$$z_0 = \frac{c_{x_0}}{r_{t_0}}\vartheta_0 \quad \text{is denoted.}$$

Figure 3 shows that

along z = const., L decreases with increasing ϑ,
along ϑ = const., L decreases with decreasing z.
(Increasing L values are marked with arrows.)

When

$$z = z_{cr} = \frac{c_A - C_e\eta}{c_w - c_v}$$

so at any values of ϑ $L = \infty$, note that $z_{cr} \gg z_0$. Further,

when $z = 0$, then $L = 0$ at any values of ϑ,
when $z = z_{cr}$, then $L = \infty$ at all values of ϑ.

This means that in the range of interest, i.e. between $z = 0$ and $z = z_0$, L changes from 0 to ∞.

It is seen that a part of the L curves, representing the values $0 \leqslant L \leqslant 1$, is in the field limited by the w.b.t. line and the ϑ axis, i.e. that of actually possible state changes.

Consequently the suggestion that the range of L in the case of heat and mass transfer processes of sorptive materials (e.g. drying) is $0 \leqslant L \leqslant 1$ has been found to be valid.

This expresses the modification and generalization of the Lewis rule for the non-steady stage of heat and mass transfer phenomena of sorptive bodies.

CONCLUSION

The mathematical relationship between the sorptive body and the ambient flowing medium is presented by the material temperature curve, the analytical determination of which has been established.

By means of the Lewis factor, L, the physical parameters of the flowing gas state as well as those of the moist sorptive material in the presence of simultaneous heat and mass transfer have been determined at any intermediate time and characterized by a new interpretation of the w.b.t. Applying the L number in the entire range of the unsteady stage, the treatment of the state changes of both the material and the accompanying gas becomes unified.

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CONTRIBUTION A LA THEORIE DES PHENOMENES DE SURFACE DES MILIEUX POREUX SOUMIS AUX TRANSFERTS DE CHALEUR ET DE MASSE

Résumé—On développe un modèle mathématique pour résoudre des problèmes de transfert de chaleur et de masse dans des milieux poreux dans un cas non permanent. L'analyse de la température de bulbe humide dans le cas permanent est déduite d'une nouvelle explication de cette température dans la période non statiminaire avec les paramètres physiques qui la détermine. Le facteur de Lewis gagne une interprétation particulière dans la période en question. On donne aussi une nouvelle interprétation de la régle de Lewis dans cette période.

BEITRAG ZUR THEORIE DER OBERFLÄCHEN-PHÄNOMENE AN PORÖSEN KÖRPERN BEI WÄRME- UND STOFFÜBERGANGSPROZESSEN

Zusammenfassung—Es wird ein mathematisches Modell zur Lösung von Wärme- und Stoffübergangsproblemen an porösen Medien im instationären Zustand entwickelt. Die Feuchtkugeltemperatur im stationären Zustand wird mittels einer neuen Definition dieser Temperatur während der instationären Periode und der sie bestimmenden physikalischen Parameter abgeleitet. Entsprechend erfährt die Lewis-Zahl eine passende Interpretation für den betreffenden Zustand. Eine neue Interpretation des Lewis'schen Gesetzes für den instationären Zustand wird ebenfalls gegeben.

К ТЕОРИИ ПОВЕРХНОСТНЫХ ЯВЛЕНИЙ ПРИ ТЕПЛО- И МАССОПЕРЕНОСЕ В ПОРИСТЫХ ТЕЛАХ

Аннотация—Разработана математическая модель для решения задач тепло-и массопереноса в пористых средах в нестационарном режиме. Анализ температуры мокрого термометра в стационарном состоянии проводится на основе нового толкования температуры мокрого термометра на нестационарной стадии процесса с учетом определяющих его физических параметров. Соответственно дается новая интерпретация коэффициента Льюиса для рассматриваемого состояния, а также новое объяснение закона Льюиса для нестационарного состояния.